

## Fluid Mechanics

It is the branch of Science which deals with the nature and properties of fluids, both in motion and at rest.

### Categories of Fluid Mechanics

There are three categories of fluid mechanics.

#### i) Fluid Statics

It deals with fluid at rest.

#### ii) Fluid Kinematics

It deals <sup>with</sup> fluid in motion without considering the force which cause or accompany the motion e.g. Speed, velocity, acceleration etc.

#### iii) Fluid Dynamics

It deals with the study of fluids in motion.

Fluid: A fluid is a substance that deforms continuously when subjected to a shear stress (i.e. tangential force), no matter how small that shear stress may be.

OR  
A fluid is a substance which is capable of ~~flowing~~ flowing.

## Characteristics:

A fluid has the following characteristics:

- ⇒ It has no definite shape of its own but conforms to the shape of the containing vessels.
- ⇒ Even a small amount of shear force exerted on a fluid will cause it to change its shape.

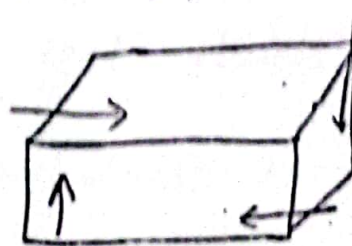
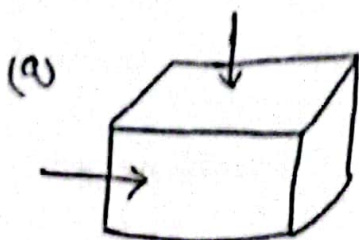
⇒ It has capability of flowing.

Note: Shearing forces are also called tangential forces.

## Fluid Properties:

A fluid has different important properties.

**Pressure:** Pressure refers to normal forces that apply to fluid particles as well as the forces that fluid applies to its container & other solid objects embedded in the fluid (Fig a).



Component of stress (a) (normal) pressure  
(b) shear.

conditions - the assumption of ideal fluid helps in simplifying the mathematical analysis.

## REAL FLUID:

A real fluid is one which has viscosity, surface tension & compressibility. The real fluids are actually available in nature.

### : Viscosity:

Viscosity is the measure of resistance of a fluid which is being deformed by either shear or tensile stress. In everyday terms (for fluids only), viscosity is thickness or internal friction. Thus water is thin having a lower viscosity, while honey is thick having a higher viscosity. Simply, the less viscous the fluid is, the greater is the ease of movement (fluidity).

Viscosity describes a fluid's internal resistance to flow & may be thought of as a measure of fluid friction. All real fluids have some resistance to shear stress & therefore are viscous, but a fluid which has no





resistance to shear stress is known as an ideal fluid or inviscid fluid.

⇒ In any flow, layers move at different velocities and fluid's viscosity arises from the shear stress b/w the layers that ultimately opposes any applied forces. The relationship b/w the shear stress & velocity gradient can be obtained by considering two plates closely spaced at a distance 'y' & separated by a homogenous substance.

Let the lower plate is fixed, & a force  $F$  be applied to the upper plate. At this force viscosity causes the substance b/w the plates to undergo shear flow with a velocity gradient " $u/y$ ", the substance is called a fluid.

The applied force is proportional to the area & velocity gradient in fluid & inversely proportional to the distance b/w the plates. Combining these three results in the equation, we have;

$$F = \mu A \frac{u}{y}$$

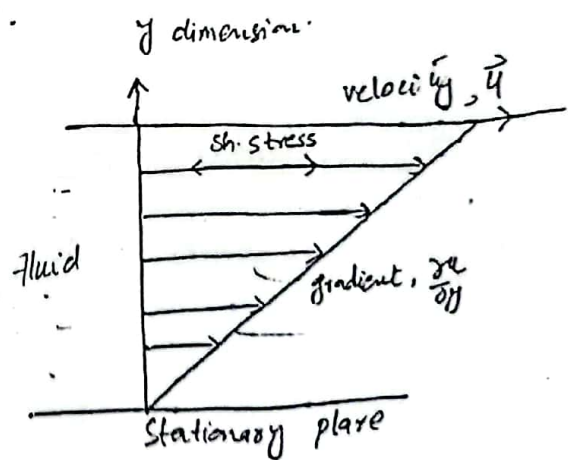
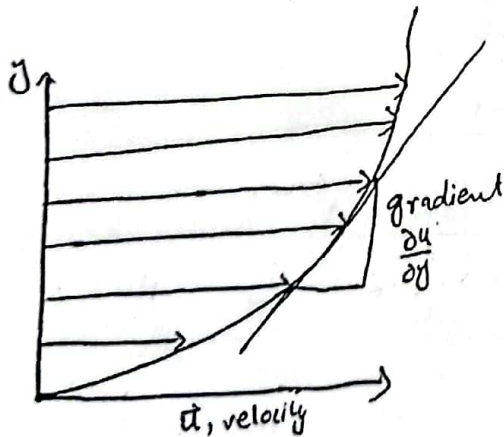
where ' $\mu$ ' is proportionality factor called dynamic viscosity.

This eq can be expressed in terms of shear stress as

$$\tau = F/A$$

Thus by 'Newton' in differential form for straight, parallel & uniform flow, the shear stress b/w layers is proportional to the velocity gradient in direction  $\perp$  to the layers.

$$\tau = \mu \frac{du}{dy}$$





## NEWTON'S LAW OF VISCOSITY:

When two layers of fluid, at a distance " $dy$ " apart move one over the other at different velocities say ' $u$ ' & ' $u+du$ '. The velocity together with the relative velocity causes a shear stress on the adjacent lower layer, while the lower layer causes the shear stress on the adjacent top layer. This shear stress of velocity w.r.t ' $y$ ' is denoted by  $T_w$  ( $\tau$ ). Mathematically

$$T_w \propto \frac{du}{dy}$$

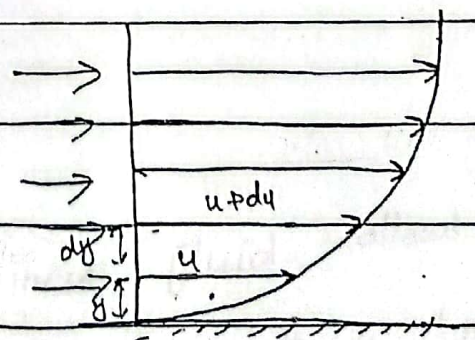
or  $T_w = \mu \frac{du}{dy}$

where ' $\mu$ ' is called constant of proportionality & is called coefficient of dynamic viscosity.

$$\mu = \frac{T_w}{du/dy}$$

$T = \text{shear stress}$

$$\mu = \frac{N/m^2}{m/s/m} = \frac{N \cdot s}{m^2}$$



$$\mu = Pa \cdot s, \quad Pa = \text{Pascal}$$
$$= \frac{N \cdot s}{m^2}$$

## Kinematic Viscosity:

$$\nu = \mu / \rho$$

$$= \frac{\text{N-s/m}^2}{\text{kg/m}^3}$$

$$\text{As } \rho = \text{kg/m}^3$$

$$\nu = \frac{\text{N-s}}{\text{m}^2} \cdot \frac{\text{m}^3}{\text{kg}}$$

$$= \frac{\text{N-s} \cdot \text{m}}{\text{kg}}$$

$$= \frac{\text{N-s}}{\text{kg/m}}$$

$$\text{As } F = ma \quad \text{For newton second law (For N)}$$

$$= \text{kg} \cdot \text{m/s}^2$$

$$\nu = \frac{\text{kg} \cdot \text{m/s}^2 \cdot \text{s}}{\text{kg/m}}$$

$$= \text{m}^2/\text{s}$$

$$\text{So } \nu = \text{m}^2/\text{s}$$

## TYPES OF FLUID:

There are two types of fluid.

(1) Newtonian fluid.

(2) Non-newtonian fluid.

**Newtonian fluid:** These fluid follows the Newton's law of viscosity, for such fluids " $\mu$ " does not change with the rate of deformation e.g. water & air.



in these fluids stress  $\propto$  strain.

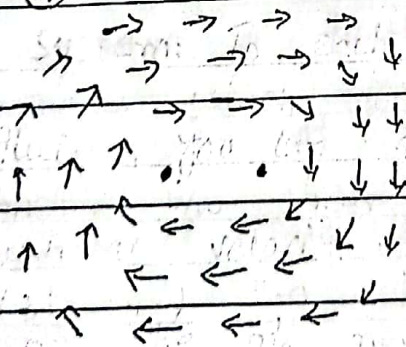
### Non-Newtonian Fluid:

These are fluids which does not follow the linear relationship b/w shear stress & rate of deformation (strain) e.g. blood, polymer solutions, mud, ketchup & honey etc.

### Discussion of Fluid Motion:

**Field based system:** To each point in a region containing fluid, you can ascribe a set of properties; such as velocity, pressure, temp, density. The position of the pts never more. This treatment of fluids as a field is called "Eulerian view"

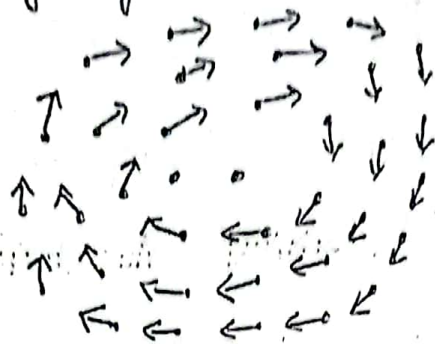
e.g. in fig (a)



**Particle based system:** You can think a fluid in terms of a vast collection of particles that move around. Each particle (or parcel) has



with the Eulerian view, where position is fixed to grid. This treatment of fluid is called Lagrangian view.



Although gases & liquids are made up of atoms & molecules, for many purposes it is useful & sufficient to think of them as continuous media that can flow. One way of describing the motion of fluid is to assign coordinates  $(x, y, z)$  to a given fluid element & then specify these coordinates as a function of time. Suppose we have a fluid element that is at position  $(x_0, y_0, z_0)$  at time  $t_0$ . At later times, the position of this element is described by

$x = x(x_0, y_0, z_0, t)$   
properties such as position, velocity, temp, density =  $y(x_0, y_0, z_0, t)$  in contrast

$$z = z(x_0, y_0, z_0, t)$$

This way of describing the fluid is originally due to Euler & leads to the lagrangian eqs of fluid dynamics.

Another way of describing fluid, also due to Euler, is to specify the fluid properties (e.g. velocity, density) at each pt in space at each instant of time. The density for example, is then specified by a function

$$\rho = \rho(x, y, z, t)$$

This picture leads to the Eulerian eqs of fluid dynamics. In the Euler picture, attention is focused on what happening at a particular pt in space, rather than on a particular fluid element.

Since these two descriptions are equivalent there are relationship b/w the lagrangian



4 Eulerian eqs of fluid motion. In Eulerian picture, the time derivative of, for example, the density is determined at a fixed pt in a space & hence is the partial derivative w.r.t 't' keeping x, y & z fixed

$$\frac{\partial \rho}{\partial t} = \lim_{\Delta t \rightarrow 0} \frac{\rho(x, y, z, t + \Delta t) - \rho(x, y, z, t)}{\Delta t}$$

In the Lagrangian picture, the time derivative is the total derivative

$$\frac{d\rho}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\rho(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) - \rho(x, y, z, t)}{\Delta t}$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} \frac{dx}{dt} + \frac{\partial \rho}{\partial y} \frac{dy}{dt} + \frac{\partial \rho}{\partial z} \frac{dz}{dt}$$

$$= \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} v_x + \frac{\partial \rho}{\partial y} v_y + \frac{\partial \rho}{\partial z} v_z$$

where

$$V = (v_x, v_y, v_z)$$

is the velocity of the fluid element. The relationship b/w the derivatives is

$$\frac{dX}{dt} = \frac{\partial X}{\partial t} + V \cdot \nabla X$$

where X is any variable describing a property of fluid.

## STEADY OR UNSTEADY FLOW:

**Steady Flow:** This type of flow in which the fluid characteristics like velocity, pressure, density etc. at a pt do not change with time, is called steady flow.

Mathematically:

$$\left. \frac{\partial u}{\partial t} \right|_{x_0, y_0, z_0} = 0$$

$$\left. \frac{\partial v}{\partial t} \right|_{x_0, y_0, z_0} = 0 \quad \left. \frac{\partial w}{\partial t} \right|_{x_0, y_0, z_0} = 0$$

$$\left. \frac{\partial p}{\partial t} \right|_{x_0, y_0, z_0} = 0 \quad \dots \dots \dots$$

**Un-Steady Flow:** The type of the flow in which the fluid characteristic like velocity, pressure, density etc change with time is called unsteady flow. Mathematically

$$\left. \frac{\partial u}{\partial t} \right|_{x_0, y_0, z_0} \neq 0, \quad \left. \frac{\partial v}{\partial t} \right|_{x_0, y_0, z_0} \neq 0$$

$$\left. \frac{\partial w}{\partial t} \right|_{x_0, y_0, z_0} \neq 0 \quad \& \quad \left. \frac{\partial p}{\partial t} \right|_{x_0, y_0, z_0} \neq 0$$

**Uni-form flow:** The type of flow in which the velocity at any give time does not



change w.r.t. space is called uniform

flow mathematically

$$\frac{\partial v}{\partial s} \Big|_{t=t_0} = 0$$

we can write similarly.  $s$  is general displacement velocity same at any pt.

### Non-Uniform Flow:

The type of the flow in which the velocity at any given time changes w.r.t space is called non-uniform flow. Mathematically

$\frac{\partial v}{\partial s} \Big|_{t=t_0} \neq 0$

**Rotational Flow:** A flow is said to be rotational flow if the fluid particles while moving in the flow direction rotate about any axes or center of mass.

**Note:** The pt at which whole forces, pressure or mass concentrate or acts is called centre of mass.

### Irr. Rotational Flow:

A flow is said to be irr-rotational if the fluid particles while moving do not rotate about

## Laminar Flow:

Laminar flow is one in which paths taken by the individual particles do not cross one another & move along well defined paths. This type of flow is called stream line flow or viscous flow e.g. flow in a circular pipe.

**Turbulent Flow:** Turbulent flow is one in which path taken by the individual particles cross one another, this type of flow is called turbulent flow.

**Compressible Flow:** That type of fluid flow in which density is not constant i.e.

$$\rho \neq \text{constant}$$

$$\frac{\partial \rho}{\partial t} \neq 0$$

**Incompressible Flow:** That type of flow in which density is constant i.e.